Discrete Math

Logic Practice Problems

Name: ....

Id: ....

Grade

1. Find all possible Truth assignments to the 4 variables x, y, z, w (out of the possible 16 assignments) that satisfy the following formula:

$$(x \vee y) \wedge (\neg y \vee z) \wedge (\neg z \vee w) \wedge (\neg w \vee \neg x)$$

2. Tautologies and Contradictions.

   (a) Prove that $(x \wedge y) \rightarrow (x \vee y)$ is a tautology.

   (b) Prove that $(x \oplus y) \wedge (x \equiv y)$ is a contradiction.

3. Prove the two logic distributive laws.

(a) $x \wedge (y \vee z) \equiv (x \wedge y) \vee (x \wedge z)$.

(b) $x \vee (y \wedge z) \equiv (x \vee y) \wedge (x \vee z)$.

4. Prove the two De Morgan's laws for four variables.

(a) $\neg(x \wedge y \wedge z \wedge w) \equiv (\neg x) \vee (\neg y) \vee (\neg z) \vee (\neg w)$.

(b) $\neg(x \vee y \vee z \vee w) \equiv (\neg x) \wedge (\neg y) \wedge (\neg z) \wedge (\neg w)$.

5. Expressing $\mathcal{NOR}$ ($\downarrow$) with $\mathcal{NAND}$ ($\uparrow$) and $\mathcal{NAND}$ ($\uparrow$) with $\mathcal{NOR}$ ($\downarrow$).

   (a) Show how to express $\mathcal{NOR}$ with $\mathcal{NAND}$. Prove that your answer is correct.

   (b) Show how to express $\mathcal{NAND}$ with $\mathcal{NOR}$. Prove that your answer is correct.

6. Prove that if $p \to q$ and $q \to r$ then $p \to r$.

7. Assume that $U = \{1, 2, 3\}$. For each of the following statements determine if it is True or False. Justify your answers.

   (a) $\exists_{x \in U} \forall_{y \in U} (x^2 < y + 1)$

   (b) $\forall_{x \in U} \forall_{y \in U} (x^2 + y^2 < 18)$

   (c) $\forall_{x \in U} \exists_{y \in U} (x^2 + y^2 < 11)$

8. Consider the following three sets

$$\begin{aligned} A &= \{2,4,6,8\} \\ B &= \{1,3,5,7\} \\ C &= \{1,2,7,8\} \end{aligned}$$

Consider the following four statements about a set S.

(p_1) $\forall_{x \in S} \{x \text{ is even}\}$

(p_2) $\exists_{x \in S} \{x \text{ is even}\}$

(p_3) $\forall_{x \in S} \{x \text{ is odd}\}$

(p_4) $\exists_{x \in S} \{x \text{ is odd}\}$

(a) For each of the three sets A, B, and C, find which of the four statements is true?

(b) Is there a finite set of integers for which at least three of the statements are true? Justify your answer.

9. Three boxes are presented to you. One contains gold while the other two are empty. Each box has a clue imprinted on it as to its contents. The clues are

   - Box 1: “The gold is not here”
   - Box 2: “The gold is not here”
   - Box 3: “The gold is in Box 2”

   Only one clue is true while the other two clues are false. Which box has the gold?

   Explain your answer.

10. There are three boxes. One box has two white balls; one box has two black balls; and one box has one white ball and one black ball. Three labels are attached to the boxes: WW (White and White); BB (Black and Black); and BW (Black and White). Unfortunately, each box is labeled with an **incorrect** label.

    You are allowed to open only one box, pick one ball at random, see its color, and put it back into the box without seeing the color of the other ball.

    Which box should you open to fix the labeling of the three boxes? Explain your answer.