Discrete Structures

Probability Practice Problems

1. Assume a **fair** coin in which the probability for flipping Heads (H) is $p(H) = 1/2$ and the probability of flipping Tails (T) is $p(T) = 1/2$. Justify your answers.

   (a) The fair coin is flipped 4 times.
      i. What is the probability that **exactly** 2 out of the 4 flips are heads?
      ii. What is the probability that **at least** 2 out of the 4 flips are heads?
      iii. What is the probability that **at most** 2 out of the 4 flips are heads?

   (b) The fair coin is flipped 5 times.
      i. What is the probability that **exactly** 3 out of the 5 flips are heads?
      ii. What is the probability that **at least** 3 out of the 5 flips are heads?
      iii. What is the probability that **at most** 3 out of the 5 flips are heads?

   (c) The fair coin is flipped $n \geq 2$ times. Write your answers as a function of n.
      i. What is the probability that **exactly** one out of the n flips is heads?
      ii. What is the probability that **at least** one out of the n flips is heads?
      iii. What is the probability that **at most** one out of the n flips is heads?

2. Assume a **biased** coin in which the probability for Heads (H) is 2/3 while the probability for Tails (T) is 1/3. Justify your answers.

   (a) The biased coin is flipped 4 times.
      i. What is the probability that **exactly** 2 out of the 4 flips are heads?
      ii. What is the probability that **at least** 2 out of the 4 flips are heads?
      iii. What is the probability that **at most** 2 out of the 4 flips are heads?

   (b) The biased coin is flipped 5 times.
      i. What is the probability that **exactly** 3 out of the 5 flips are heads?
      ii. What is the probability that **at least** 3 out of the 5 flips are heads?
      iii. What is the probability that **at most** 3 out of the 5 flips are heads?

   (c) The biased coin is flipped $n \geq 2$ times. Write your answers as a function of n.
      i. What is the probability that **exactly** one out of the n flips is heads?
      ii. What is the probability that **at least** one out of the n flips is heads?
      iii. What is the probability that **at most** one out of the n flips is heads?

3. Justify your answers to the following questions. By definition, when flipping the fair coin $p(H) = p(T) = 1/2$ and when flipping the biased coin $p(H) = 2/3$ and $p(T) = 1/3$.

   (a) The fair coin is flipped 4 times. What is the probability that there are no consecutive heads and no consecutive tails?

   (b) The fair coin is flipped 5 times. What is the probability that there are no consecutive heads and no consecutive tails?

   (c) The fair coin is flipped $n \geq 1$ times. What is the probability, as a function of n, that there are no consecutive heads and no consecutive tails?

   (d) The biased coin is flipped 4 times. What is the probability that there are no consecutive heads and no consecutive tails?

   (e) The biased coin is flipped 5 times. What is the probability that there are no consecutive heads and no consecutive tails?

   (f) The biased coin is flipped $n \geq 1$ times. What is the probability, as a function of n, that there are no consecutive heads and no consecutive tails?
   Remark: The answers are different for odd n and even n.

4. In a fair 6-face dice, with the numbers $1, 2, 3, 4, 5, 6$ on its faces, the probability of throwing any of the 6 numbers is $1/6$. Justify your answers.

   (a) What is the probability of throwing exactly two 6 when three dice are thrown together?

   (b) What is the probability of throwing exactly two 6 when four dice are thrown together?

   (c) What is the probability, as a function of n, of throwing exactly two 6 when $n \geq 2$ dice are thrown together?

   (d) What is the probability of throwing at least one 6 when three dice are thrown together?

   (e) What is the probability of throwing at least one 6 when four dice are thrown together?

   (f) What is the probability, as a function of n, of throwing at least one 6 when $n \geq 1$ dice are thrown together?

5. In a fair 5-face dice, with the numbers $1, 2, 3, 4, 5$ on its faces, the probability of throwing any of the 5 numbers is $1/5$. Justify your answers.

   (a) What is the probability of throwing exactly two 5 when three dice are thrown together?

   (b) What is the probability of throwing exactly two 5 when four dice are thrown together?

   (c) What is the probability, as a function of n, of throwing exactly two 5 when $n \geq 2$ dice are thrown together?

   (d) What is the probability of throwing at least one 5 when three dice are thrown together?

   (e) What is the probability of throwing at least one 5 when four dice are thrown together?

   (f) What is the probability, as a function of n, of throwing at least one 5 when $n \geq 1$ dice are thrown together?

6. A deck of cards contains 52 cards. There are 4 suits: 13 Black Clubs, 13 Red Diamonds, 13 Red Hearts, and 13 Black Spades. Each suit has one of the following 9 number cards: $2, 3, 4, 5, 6, 7, 8, 9, 10$, one of the following 3 face cards: Jack (J), Queen (Q), and King (K), and one Ace (A). A bridge hand has 13 random cards out of the 52 cards

   Remark: There are $\binom{52}{13}$ different bridge hands. Define $x = \binom{52}{13}$. In your answers, you may use x and define other variables for other large numbers or other large binomial coefficients.

   Justify your answers.

   (a) What is the probability that all the 13 cards in a bridge hand are of the same suit?

   (b) What is the probability that all the 13 cards in a bridge hand are red?

   (c) What is the probability that a bridge hand contains only faces and aces?

   (d) What is the probability that a bridge hand contains only number cards 2 to 10?

   (e) What is the probability that a bridge hand does not contain an ace?

   (f) What is the probability that a bridge hand contains all 4 aces?

   (g) What is the probability that a bridge hand does not contain two of a kind?

7. A bag contains 16 marbles: 3 Blue (B) marbles, 5 Red (R) marbles, and 8 Green (G) marbles.

   Justify your answers.

   (a) After drawing a marble from the bag the marble is put aside.

      i. What is the probability that 2 drawn marbles are of the same color?
      ii. What is the probability that 2 drawn marbles are of different colors?
      iii. What is the probability that 3 drawn marbles are of the same color?
      iv. What is the probability that 3 drawn marbles are of different colors?

   (b) After drawing a marble from the bag the marble is put back in the bag.

      i. What is the probability that 2 drawn marbles are of the same color?
      ii. What is the probability that 2 drawn marbles are of different colors?
      iii. What is the probability that 3 drawn marbles are of the same color?
      iv. What is the probability that 3 drawn marbles are of different colors?
      v. What is the probability, as a function of n, that $n \geq 2$ drawn marbles are of the same color?

8. There are two bags of marbles. Each bag contains 6 marbles. The first contains 1 Red marble, 2 Blue marbles, and 3 Green marbles while the second contains 3 Red marbles, 2 Blue marbles, and 1 Green marble. You draw 1 random marble from each bag.

   Justify your answers.

   (a) What is the probability that both marbles are Red?

   (b) What is the probability that at least one marble is Red?

   (c) What is the probability that both marbles are Blue?

   (d) What is the probability that at least one marble is Blue?

   (e) What is the probability that both marbles are Green?

   (f) What is the probability that at least one marble is Green?

   (g) What is the probability that both marbles have the same color?

   (h) What is the probability that both marbles have the same color given that none of the marbles is Red?

   (i) What is the probability that both marbles have the same color given that none of the marbles is Blue?

   (j) What is the probability that both marbles have the same color given that none of the marbles is Green

   (k) What is the probability that the two marbles have different colors?

   (l) What is the probability that the two marbles have different colors given that none of the marbles is Red?

   (m) What is the probability that the two marbles have different colors given that none of the marbles is Blue?

   (n) What is the probability that the two marbles have different colors given that none of the marbles is Green?

9. The six faces of a fair 6-face dice are numbered with $1, 2, 2, 3, 3, 3$ (instead of $1, 2, 3, 4, 5, 6$). As a result, a single throw of this dice shows 1 with probability $1/6$, shows 2 with probability $2/6 = 1/3$, and shows 3 with probability $3/6 = 1/2$. This dice is thrown twice.

   Justify your answers.

   (a) What is the probability that the sum of both throws is 5?

   (b) What is the probability that the sum of both throws is 5 given that none of them is 1?

   (c) What is the probability that the product of both throws is 3?

   (d) What is the probability that the product of both throws is 3 given that none of them is 2?

   (e) What is the probability that the sum of both throws is even?

   (f) What is the probability that the sum of both throws is even given that at least one throw shows 1?

   (g) What is the probability that the sum of both throws is even given that at least one throw shows 2?

   (h) What is the probability that the sum of both throws is even given that at least one throw shows 3?

   (i) What is the probability that the sum of both throws is odd?

   (j) What is the probability that the sum of both throws is odd given that at least one throw shows 1?

   (k) What is the probability that the sum of both throws is odd given that at least one throw shows 2?

   (l) What is the probability that the sum of both throws is odd given that at least one throw shows 3?