Discrete Math

Sets Practice Problems

Name: .....

Id: .....

Grade

1. Let U be the set of all the positive integers smaller than 19, let A be the set of all the even positive integers smaller than 19, and let B be the set of all the positive integers smaller than 19 that are divided by 3. Find the members of each of the following 11 sets. The complement of sets should be defined relative to the set U which contains both A and B.
   (a) $A =$ _____
   (b) $B =$ _____
   (c) $\neg A =$ _____
   (d) $\neg B =$ _____
   (e) $A \cup B =$ _____
   (f) $A \cap B =$ _____
   (g) $A \setminus B =$ _____
   (h) $B \setminus A =$ _____
   (i) $A \triangle B =$ _____
   (j) $\neg(A \cup B) =$ _____
   (k) $\neg(A \cap B) =$ _____

2. Express the two De Morgan's laws for the four sets A, B, C, and D in four ways using the four negation notations for a set X: X', $\overline{X}$, $\neg X$, and X^C.

3. Write the principle of inclusion exclusion for the five sets A, B, C, D, and E.

4. Prove that the distributive laws for sets are correct. You may use Venn diagrams.

   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. Prove the following two statements:

   (a) If $A \subseteq B$ and $A \subseteq C$ then $A \subseteq (B \cap C)$

   (b) If $A \subseteq C$ and $B \subseteq C$ then $(A \cup B) \subseteq C$

6. Prove or give a counterexample for each of the following two statements:

   (a) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

   (b) If $A \in B$ and $B \in C$ then $A \in C$

7. Let A and B be two sets. If $2^A \subseteq 2^B$, what is the relation between A and B? (For a set X, the set 2^X is the set of all the subsets of X.) Justify your answer.

8. Prove or disprove: for any three sets A, B, and C: $C \setminus (A \cap B) = (C \setminus A) \cap (C \setminus B)$

9. Let $a(t) \leq b(t) \leq c(t)$ be the lengths of the three sides of a triangle t in a non-decreasing order. Define the following sets:
   - T: The set of all triangles.
   - X: The set of all triangles t for which $a(t) = b(t)$.
   - Y: The set of all triangles t for which $b(t) = c(t)$.

   Using only set operations on these three set, define the following sets. Justify your answers.

   (a) The set of all equilateral triangles (all sides equal).

   (b) The set of all isosceles triangles (at least two sides equal).

   (c) The set of all scalene triangles (no two sides equal).

10. For two positive integers $m < n$, let A be a set with m elements ($|A| = m$) and B a set with n elements ($|B| = n$). For each of the following sets, give upper and lower bounds on their cardinality.

    An upper bound states the maximum possible elements in the set. A lower bound states the minimum possible elements in the set.

    Explain your answers.

    (a) $A \cap B$

    (b) $A \cup B$

    (c) $A \setminus B$

    (d) $B \setminus A$

11. There are 50 students in a class. 30 students study Math, 25 students study Computer Science, and 15 students study both Math and Computer Science.

    (a) How many students study only Math? _____

    (b) How many students study only Computer Science? _____

    (c) How many students study neither Math nor Computer Science? _____

    Explain how you found the answers.

12. 36 students take the Discrete Structures class. They had 3 quizzes called X, Y, and Z.
    - Surprisingly, only 1 student did not get an A on any quiz while the rest of the students got an A (*aced*) on at least one quiz.
    - 21 students aced X, 19 students aced Y, and 17 students aced Z.
    - 9 students aced both X and Y, 11 students aced both X and Z, and 8 students aced both Y and Z.

    (a) How many students aced at least one quiz? _____
    (b) How many students aced all three quizzes? _____
    (c) How may students aced both X and Y but not Z? _____
    (d) How may students aced both X and Z but not Y? _____
    (e) How may students aced both Y and Z but not X? _____
    (f) How may students aced only X? _____
    (g) How may students aced only Y? _____
    (h) How may students aced only Z? _____

    Justify your answers.