Definition 1. [Even integers] *An integer n is called* ***even*** *if there exists an integer k such that $n = 2k$.*

Theorem 1. [Sum of even integers] *If m and n are both even integers, then their sum, $m + n$, is also even.*

Proof. [Sum of even integers] Let m and n be two even integers. According to the definition of even integers, there exists $k_1 \in \mathbb{Z}$ such that $m = 2k_1$ and $k_2 \in \mathbb{Z}$ such that $n = 2k_2$.

Notice that

$$m + n = 2k_1 + 2k_2 = 2(k_1 + k_2),$$

which is, by definition, an even integer because $(k_1 + k_2)$ is an integer. $\square$

Corollary 1. [Square of an odd integer] *If n is an odd integer, then n^2 is also odd.*

This is a corollary of a probably more general theorem:

Theorem 2. [Product of odd integers] *If $n_1, n_2, n_3, \ldots, n_m$ are odd integers, then their product, $n_1 \cdot n_2 \cdot n_3 \cdot \cdots \cdot n_m$, is also odd.*