The Complexity of Online Bribery in Sequential Elections

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My thanks to the organizers. The slides are thanks to Jörg (+ minor changes).

The 20-21 New York Colloquium on Algorithms and Complexity

(virtually) New York, NY, March 19, 2021


HOUSE ARRYN
AS HIGH AS HONOUR
HOUSE MORMONT
HERE WE STAND
HOUSE GREYJOY
WE DO NOT SOW
HOUSE CLEGANE
HOUSE LANNISTER
HEAR ME ROAR
HOUSE STARK
WINTER IS COMING
HOUSE TYRELL
GROWING STRONG
HOUSE TARGARYEN
FIRE AND BLOOD
HOUSE BARATHEON
HOUSE TULLY
FAMILY, DUTY, HONOUR

Elections and Voting Rules

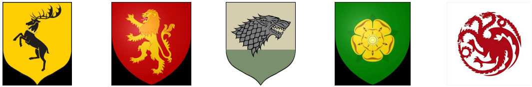
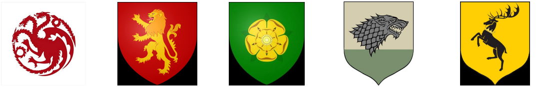
Scoring Protocol: Plurality

$$\alpha = (1, 0, 0, 0, 0)$$

Voter 1:					
Voter 2:					
Voter 3:					
Voter 4:					

Elections and Voting Rules

Scoring Protocol: Plurality

$\alpha \;=$	$(1, \quad 0, \quad 0, \quad 0, \quad 0)$
Voter 1:	
Voter 2:	
Voter 3:	
Voter 4:	

Elections and Voting Rules

Scoring Protocol: Borda

α	$=$	$(4,$	$3,$	$2,$	$1,$	$0)$
Voter 1:						
Voter 2:						
Voter 3:						
Voter 4:						

Elections and Voting Rules

Scoring Protocol: Borda

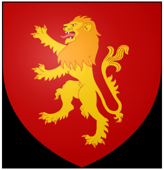
$$\alpha = (4, \quad 3, \quad 2, \quad 1, \quad 0)$$

Voter 1:

Voter 2:

Voter 3:

Voter 4:

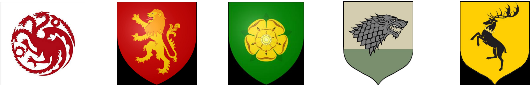


Elections and Voting Rules

Scoring Protocol: Veto

α =	(1,	1,	1,	1,	0)

Voter 1: 

Voter 2: 

Voter 3: 

Voter 4: 

Elections and Voting Rules

Scoring Protocol: Veto

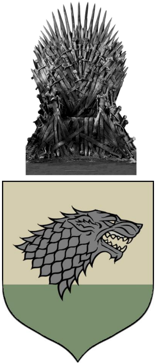
$$\alpha = (1, \quad 1, \quad 1, \quad 1, \quad 0)$$

Voter 1:

Voter 2:

Voter 3:

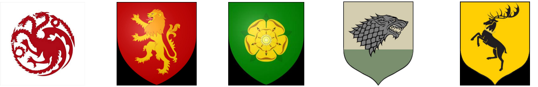
Voter 4:



Manipulation

Scoring Protocol: Borda

$$\alpha = (4, \quad 3, \quad 2, \quad 1, \quad 0)$$

Voter 1:	
Voter 2:	
Voter 3:	
Voter 4:	

Manipulation

Scoring Protocol: Borda

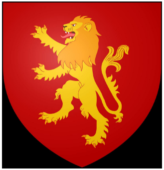
$\alpha =$	$(4,$	$3,$	$2,$	$1,$	$0)$
Voter 1:					
Voter 2:					
Voter 3:					
Voter 4:					

Control

Scoring Protocol: Borda

$\alpha = (4, 3, 2, 1, 0)$

Voter 1:					
Voter 2:					
Voter 3:					
Voter 4:					

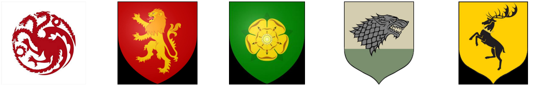


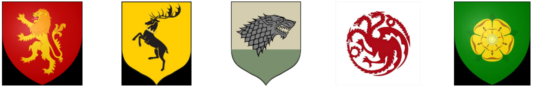
Control by Deleting Voters

Scoring Protocol: Borda

$$\alpha = (4, \quad 3, \quad 2, \quad 1, \quad 0)$$

~~Voter 1:~~

Voter 2: 

Voter 3: 

Voter 4: 

Bribery

Scoring Protocol: Borda

$$\alpha = (4, \quad 3, \quad 2, \quad 1, \quad 0)$$

Voter 1: ($ 1, weight 1)					
Voter 2: ($ 1, weight 1)					
Voter 3: ($ 1, weight 1)					
Voter 4: ($ 1, weight 1)					

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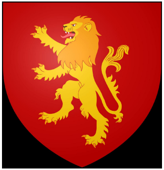

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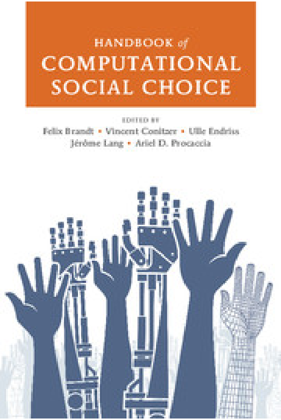
Bribery

Scoring Protocol: Borda

$\alpha =$	(4,	3,	2,	1,	0)
Voter 1: ($ 1, weight 1)					
Voter 2: ($ 1, weight 1)					
bribed Voter 3: ($ 1, weight 1)					
Voter 4: ($ 1, weight 1)					

More Details, Motivating Scenarios, Results, . . . in



Online Manipulation in Sequential Elections

Scoring Protocol: Borda

$$\alpha = (4, \quad 3, \quad 2, \quad 1, \quad 0)$$

Voter 1:
(manipulator)

Voter 2:

Voter 3:

Voter 4:
(manipulator)

E. Hemaspaandra, L. Hemaspaandra & J. Rothe:
The Complexity of Online Manipulation of Sequential Elections, JCSS 2014

Online Manipulation in Sequential Elections

Scoring Protocol: Borda

$\alpha = (4, 3, 2, 1, 0)$

Voter 1:
(manipulator)

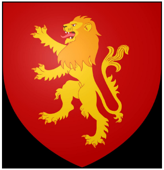
    

Voter 2:

Voter 3:

Voter 4:
(manipulator)



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Online Manipulation in Sequential Elections

Scoring Protocol: Borda

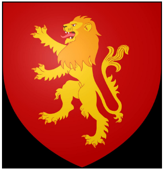
$\alpha = (4, 3, 2, 1, 0)$

Voter 1:
(manipulator)

Voter 2:

Voter 3:

Voter 4:
(manipulator)



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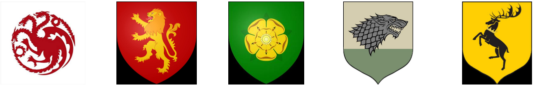
Online Manipulation in Sequential Elections

Scoring Protocol: Borda

$\alpha = (4, \quad 3, \quad 2, \quad 1, \quad 0)$

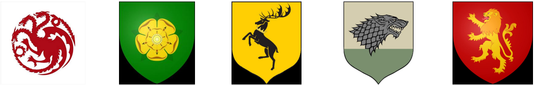
Voter 1: (manipulator)

Voter 2:



Voter 3:

Voter 4: (manipulator)



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Online Manipulation in Sequential Elections

Scoring Protocol: Borda

$\alpha = (4, 3, 2, 1, 0)$

Voter 1: (manipulator)

Voter 2:

Voter 3:

Voter 4: (manipulator)

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Online Control in Sequential Elections

Scoring Protocol: Borda

$$\alpha = (4, \quad 3, \quad 2, \quad 1, \quad 0)$$

Voter 1:     

Voter 2:

Voter 3:

Voter 4:

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The Complexity of Online Voter Control in Sequential Elections, JAAMAS 2017

Online Control in Sequential Elections

Scoring Protocol: Borda

$\alpha = (4, 3, 2, 1, 0)$

~~Voter 1:~~

Voter 2:

Voter 3:

Voter 4:

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Online Control in Sequential Elections

Scoring Protocol: Borda

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Voter 1:     

Voter 2:

Voter 3:

Voter 4:

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Online Control in Sequential Elections

Scoring Protocol: Borda

$\alpha = (4, 3, 2, 1, 0)$

Voter 1:					
Voter 2:					
Voter 3:					
Voter 4:					

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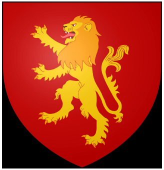

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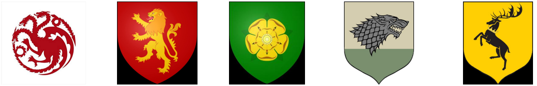
Voter 2: 

Voter 3: 

Voter 4:

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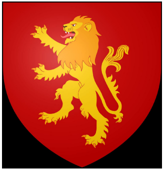

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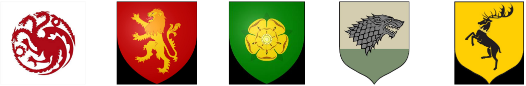
Voter 2: 

~~Voter 3:~~

Voter 4:

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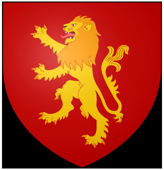

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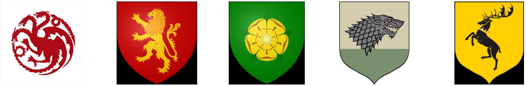
Voter 2: 

~~Voter 3:~~

Voter 4: 

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Online Bribery in Sequential Elections

Scoring Protocol: Borda

$\alpha = (4, 3, 2, 1, 0)$

Voter 1: ($ 1, weight 1)     

Voter 2: ($ 1, weight 1)

Voter 3: ($ 1, weight 1)

Voter 4: ($ 1, weight 1)

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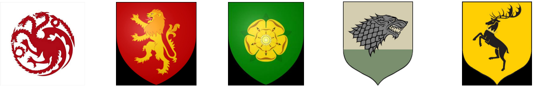
Online Bribery in Sequential Elections

Scoring Protocol: Borda

$\alpha = (4, 3, 2, 1, 0)$

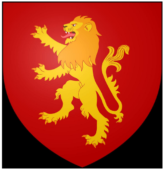
Voter 1:
($ 1, weight 1)

Voter 2:
($ 1, weight 1)



Voter 3:
($ 1, weight 1)

Voter 4:
($ 1, weight 1)



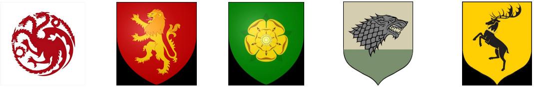
Online Bribery in Sequential Elections

Scoring Protocol: Borda

$\alpha = (4, 3, 2, 1, 0)$

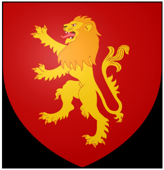
Voter 1:
($ 1, weight 1)

Voter 2:
($ 1, weight 1)



Voter 3:
($ 1, weight 1)

Voter 4:
($ 1, weight 1)



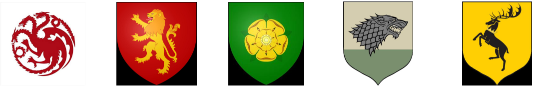
Online Bribery in Sequential Elections

Scoring Protocol: Borda

$\alpha = (4, \quad 3, \quad 2, \quad 1, \quad 0)$

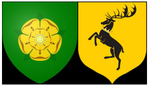
Voter 1:
($ 1, weight 1)

Voter 2:
($ 1, weight 1)



bribed
Voter 3:
($ 1, weight 1)

Voter 4:
($ 1, weight 1)



Online Bribery in Sequential Elections

Scoring Protocol: Borda

α =	(4,	3,	2,	1,	0)
Voter 1: ($ 1, weight 1)					
Voter 2: ($ 1, weight 1)					
bribed Voter 3: ($ 1, weight 1)					
Voter 4: ($ 1, weight 1)					

Online Bribery in Sequential Elections

- An *online bribery setting (OBS)* is a 5-tuple (C, V, σ, d, k), where
  - C is a set of candidates;
  - $V = (V_{<u}, u, V_{u<})$ is an election snapshot for C and u;
  - σ is the preference order of the briber;
  - $d \in C$ is a distinguished candidate; and
  - k is a nonnegative integer (the budget).

Online Bribery in Sequential Elections

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  - C is a set of candidates;
  - $V = (V_{<u}, u, V_{u<})$ is an election snapshot for C and u;
  - σ is the preference order of the briber;
  - $d \in C$ is a distinguished candidate; and
  - k is a nonnegative integer (the budget).

- If u is a voter and C is a candidate set, an *election snapshot for C and u* is specified as $V = (V_{<u}, u, V_{>u})$, where
  - $V_{<u}$ are the previous voters, in order, with their votes and information on whether they were bribed (and, if appropriate, at what cost);
  - u is the current voter, with her (unless-we-bribe-her) vote; and
  - $V_{>u}$ is simply a list, in the order they will vote, of the voters after u,

each with or without prices and/or weights, depending on the model.

Online Bribery in Sequential Elections

online-$\mathscr{E}$-Bribery

Given: An OBS (C, V, σ, d, k).

Question: Does there exist a legal choice by the briber on whether to bribe u and, if the choice is to bribe, of what vote to bribe u into casting, such that if the briber makes that choice then no matter what votes the remaining voters after u are (later) revealed to have, the **briber's goal** can be reached by the current decision regarding u and by using the briber's future (legal-only, of course) decisions (if any), each being made using the briber's then-in-hand knowledge about what votes have been cast by then?

Online Bribery in Sequential Elections

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The briber's goal: $W_{\mathscr{E}}(C, U) \cap \{c \mid c \geq_\sigma d\} \neq \emptyset$, where U are the votes after this process and $W_{\mathscr{E}}(C, U)$ is the winner set of $\mathscr{E}$ election (C, U).

Online Bribery in Sequential Elections

Variations of online bribery problems (with no limits on the number of bribes):

- online-$\mathscr{E}$-Bribery: as above (constructive, unpriced, unweighted),
- online-$\mathscr{E}$-Destructive-Bribery: $W_{\mathscr{E}}(C,U) \cap \{c \mid d \geq_\sigma c\} = \emptyset$,
- online-$\mathscr{E}$-\$Bribery: constructive, priced, unweighted,
- online-$\mathscr{E}$-Destructive-\$Bribery: destructive, priced, unweighted,
- online-$\mathscr{E}$-Weighted-Bribery,
- online-$\mathscr{E}$-Destructive-Weighted-Bribery,
- online-$\mathscr{E}$-Weighted-\$Bribery, and
- online-$\mathscr{E}$-Destructive-Weighted-\$Bribery.

Online Bribery in Sequential Elections

Variations of online bribery problems
(with limits on the number of bribes):

- online-$\mathscr{E}$-Bribery[k]: as above (constructive, unpriced, unweighted),
- online-$\mathscr{E}$-Destructive-Bribery[k]: $W_{\mathscr{E}}(C,U) \cap \{c \mid d \geq_\sigma c\} = \emptyset$,
- online-$\mathscr{E}$-\$Bribery[k]: constructive, priced, unweighted,
- online-$\mathscr{E}$-Destructive-\$Bribery[k]: destructive, priced, unweighted,
- online-$\mathscr{E}$-Weighted-Bribery[k],
- online-$\mathscr{E}$-Destructive-Weighted-Bribery[k],
- online-$\mathscr{E}$-Weighted-\$Bribery[k], and
- online-$\mathscr{E}$-Destructive-Weighted-\$Bribery[k].

General Upper Bound: No Limits on Number of Bribes

Theorem

1. *For each election system $\mathscr{E}$ whose winner problem in the unweighted case is in polynomial time (or even in polynomial space), each of*
online-$\mathscr{E}$-Bribery,
online-$\mathscr{E}$-Destructive-Bribery,
online-$\mathscr{E}$-\$Bribery, *and*
online-$\mathscr{E}$-Destructive-\$Bribery *is in* PSPACE.

General Upper Bound: No Limits on Number of Bribes

Theorem

1. *For each election system $\mathscr{E}$ whose winner problem in the unweighted case is in polynomial time (or even in polynomial space), each of* online-$\mathscr{E}$-Bribery, online-$\mathscr{E}$-Destructive-Bribery, online-$\mathscr{E}$-\$Bribery, *and* online-$\mathscr{E}$-Destructive-\$Bribery *is in* PSPACE.

2. *For each election system $\mathscr{E}$ whose winner problem in the weighted case is in polynomial time (or even in polynomial space), each of* online-$\mathscr{E}$-Weighted-Bribery, online-$\mathscr{E}$-Destructive-Weighted-Bribery, online-$\mathscr{E}$-Weighted-\$Bribery, *and* online-$\mathscr{E}$-Destructive-Weighted-\$Bribery *is in* PSPACE.

A Result about Alternating Turing Machines

Definition

The *weight of a path ρ in the tree of an ATM* is its number of maximal existential segments such that the concatenation of the bits guessed in that segment is not the 1-bit string 0.

A Result about Alternating Turing Machines

Definition

The *weight of a path ρ in the tree of an ATM* is its number of maximal existential segments such that the concatenation of the bits guessed in that segment is not the 1-bit string 0.

Theorem

Let $k \geq 0$ be fixed. Each polynomial-time ATM M such that on no input does M have an accepting path of weight strictly greater than k accepts a language in Π^p_{2k+1}.

A Result about Alternating Turing Machines

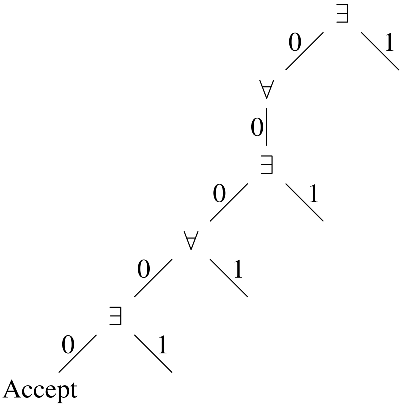

Figure: A weight 0 path in the tree of an ATM.

General Upper Bound: With Limits on Number of Bribes

Theorem

1. *For each $k \in \{0, 1, 2, \ldots\}$, and for each election system $\mathcal{E}$ whose winner problem in the unweighted case is in polynomial time, each of*
   online-$\mathcal{E}$-Bribery[k],
   online-$\mathcal{E}$-Destructive-Bribery[k],
   online-$\mathcal{E}$-\$Bribery[k], *and*
   online-$\mathcal{E}$-Destructive-\$Bribery[k] *is in* Π^p_{2k+1}.

General Upper Bound: With Limits on Number of Bribes

Theorem

1. *For each $k \in \{0, 1, 2, \ldots\}$, and for each election system $\mathscr{E}$ whose winner problem in the unweighted case is in polynomial time, each of* online-$\mathscr{E}$-Bribery$[k]$, online-$\mathscr{E}$-Destructive-Bribery$[k]$, online-$\mathscr{E}$-\$Bribery$[k]$, *and* online-$\mathscr{E}$-Destructive-\$Bribery$[k]$ *is in* Π_{2k+1}^{p}.

2. *For each $k \in \{0, 1, 2, \ldots\}$, and for each election system $\mathscr{E}$ whose winner problem in the weighted case is in polynomial time, each of* online-$\mathscr{E}$-Weighted-Bribery$[k]$, online-$\mathscr{E}$-Destructive-Weighted-Bribery$[k]$, online-$\mathscr{E}$-Weighted-\$Bribery$[k]$, *and* online-$\mathscr{E}$-Destructive-Weighted-\$Bribery$[k]$ *is in* Π_{2k+1}^{p}.

Matching Lower Bounds: No Limits on Number of Bribes

Theorem

1. *For each problem $\mathfrak{B}$ from this list of problems:*
   online-$\mathscr{E}$-Bribery,
   online-$\mathscr{E}$-Destructive-Bribery,
   online-$\mathscr{E}$-\$Bribery,
   online-$\mathscr{E}$-Destructive-\$Bribery,
   online-$\mathscr{E}$-Weighted-Bribery,
   online-$\mathscr{E}$-Destructive-Weighted-Bribery,
   online-$\mathscr{E}$-Weighted-\$Bribery, *and*
   online-$\mathscr{E}$-Destructive-Weighted-\$Bribery,
   there exists an (unweighted) election system $\mathscr{E}$, whose winner problem in both the unweighted case and the weighted case is in polynomial time, such that $\mathfrak{B}$ is PSPACE*-complete.*

Matching Lower Bounds: With Limits on Number of Bribes

Theorem

2. *For each* $k \in \{0, 1, 2, \ldots\}$*, and for each problem* $\mathfrak{B}$ *from this list:*
   online-$\mathscr{E}$-Bribery[k],
   online-$\mathscr{E}$-Destructive-Bribery[k],
   online-$\mathscr{E}$-$Bribery[$k$],
   online-$\mathscr{E}$-Destructive-$Bribery[$k$],
   online-$\mathscr{E}$-Weighted-Bribery[k],
   online-$\mathscr{E}$-Destructive-Weighted-Bribery[k],
   online-$\mathscr{E}$-Weighted-$Bribery[$k$], *and*
   online-$\mathscr{E}$-Destructive-Weighted-$Bribery[$k$],
   there exists an (unweighted) election system $\mathscr{E}$*, whose winner problem in both the unweighted case and the weighted case is in polynomial time, such that* $\mathfrak{B}$ *is* Π_{2k+1}^{p}*-complete.*

Manipulation versus Bribery

Theorem

1. *("Regular") manipulation reduces to corresponding online bribery.*
2. *Constructive manipulation in the unique winner model reduces to corresponding online destructive bribery.*
3. *Online manipulation reduces to corresponding online priced bribery.*

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Observation

For unpriced, unweighted online bribery, it is always optimal to bribe the last k voters (we don't even have to handle u in a special way). This implies that unpriced, unweighted online bribery is certainly reducible to unweighted online manipulation, and so we inherit those upper bounds.

Plurality

Theorem

online-Plurality-Bribery, online-Plurality-Destructive-Bribery, online-Plurality-Weighted-Bribery, online-Plurality-Destructive-Weighted-Bribery, online-Plurality-\$Bribery, *and* online-Plurality-Destructive-\$Bribery *are in* P.

Plurality

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online-Plurality-\$Bribery, *and*
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Theorem

online-Plurality-Weighted-\$Bribery *and*
online-Plurality-Destructive-Weighted-\$Bribery
are NP*-complete, even when restricted to two candidates.*

Proof that online-Plurality-Weighted-$Bribery is NP-hard

We reduce from the standard NP-complete problem

Partition	
Given:	A sequence $s_1, \ldots, s_n$ of nonnegative integers with $\sum_{i=1}^{n} = 2S$.
Question:	Is there a subset $A \subseteq \{1, \ldots, n\}$ such that $\sum_{i \in A} s_i = \sum_{i \notin A} s_i$?

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Map an instance of Partition to OBS (C, V, σ, d, k), where

- $C = \{d, c\}$,
- $d >_{\sigma} c$,
- the price and weight of the ith voter are both s_i,
- u is the first voter and votes for c, and
- $k = S$.

❑

Scoring Protocols

Theorem

For each scoring vector $\alpha = (\alpha_1, \ldots, \alpha_m)$,

1. online-α-Weighted-\$Bribery *and*
   online-α-Destructive-Weighted-\$Bribery
   are in P *if* $\alpha_1 = \alpha_m$ *and* NP*-hard otherwise;

Scoring Protocols

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1. online-α-Weighted-\$Bribery *and* online-α-Destructive-Weighted-\$Bribery *are in* P *if* $\alpha_1 = \alpha_m$ *and* NP-*hard otherwise;*
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2. online-α-Weighted-Bribery *and* online-α-Destructive-Weighted-Bribery *are in* P *if* $\alpha_2 = \alpha_m$ *and* NP-*hard otherwise; and*
3. online-α-Bribery, online-α-Destructive-Bribery, online-α-$Bribery, *and* online-$\alpha$-Destructive-$Bribery *are in* P.

3-candidate-Veto

Theorem

1. online-3-candidate-Veto-Bribery,
   online-3-candidate-Veto-Destructive-Bribery,
   online-3-candidate-Veto-$Bribery, *and*
   online-3-candidate-Veto-Destructive-$Bribery *are in* P.

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2. online-3-candidate-Veto-Destructive-Weighted-Bribery *and* online-3-candidate-Veto-Weighted-Bribery *are* $\mathrm{P}^{\mathrm{NP}[1]}$*-complete.*

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3. online-3-candidate-Veto-Weighted-\$Bribery *and* online-3-candidate-Veto-Destructive-Weighted-\$Bribery *are* $\mathrm{P}^{\mathrm{NP}[1]}$*-hard and in* P^{NP} *(and we conjecture that they are* P^{NP}*-complete).*

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2. online-Approval-Weighted-\$Bribery *and* online-Approval-Destructive-Weighted-\$Bribery *are each* NP*-complete.*

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- Can we close the complexity gap between $\mathrm{P}^{\mathrm{NP}[1]}$-hardness and membership in P^{NP} for online-3-candidate-Veto-Weighted-$Bribery and online-3-candidate-Veto-Destructive-Weighted-$Bribery?

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- Study online bribery for further natural voting rules!

Conclusions and Open Questions

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The BIG Open Question!

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Open Research Issue!

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